M-theory from E8

Cite as: AIP Conference Proceedings **2150**, 030010 (2019); https://doi.org/10.1063/1.5124599
Published Online: 03 September 2019

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Abstract. Bars, Sezgin and Nishino have proposed super Yang-Mills theory in $D = 12$ and beyond. Here we consider M-theory in signature (10,1) as a reduction of signature (10,2), using a projection of $\mathfrak{e}_{8(-24)}$ along b_2. The projection gives a visual decomposition of $\mathfrak{e}_{8(-24)}$ that makes the vector of $\mathfrak{so}(10,2)$ and its spinor manifest. Reduction along a time direction yields an $\mathfrak{so}(10,1)$ invariant subspace that we associate with the spacetime of $D = 11$ M-theory.

INTRODUCTION

The study of super Yang-Mills beyond $D = 11$ with multiple time degrees of freedom was undertaken by Bars, Sezgin and Nishino [1, 2, 6]. Such higher dimensional Yang-Mills theories are especially of interest for quantum gravity in that $D = 10$ super Yang-Mills is used for a matrix formulation of M-theory [7].

It was noticed the algebraic structure of super Yang-Mills in $D = 10, 12, 14$ is captured by the exceptional Lie algebras $\mathfrak{e}_{6(-26)}$, $\mathfrak{e}_{7(-25)}$, and $\mathfrak{e}_{8(-24)}$, respectively [8]. Such Yang-Mills theories were dubbed *exceptional Yang-Mills theories* and defined to arbitrary dimension using the novel mathematical framework of *exceptional periodicity* [5] and its *magic star* projections along a_2 [3, 4, 5]. In this analysis, we use a projection of $\mathfrak{e}_{8(-24)}$ along b_2 to show the vector and spinor of M-theory in $D = 10 + 2$ can be recovered. Algebraically, this is tantamount to splitting the fundamental **56** of $\mathfrak{e}_{7(-25)}$ as a Freudenthal triple system into $\mathbf{12} \oplus \mathbf{32} \oplus \mathbf{12}$. By nullifying a timelike coordinate of the **12** one recovers an $\mathfrak{so}(10,1)$ invariant subspace we associate with the total space of $D = 11$ M-theory.

M-THEORY

M-theory in $D = 10 + 1$ can be studied as a Yang-Mills theory starting with a 3-form $C_{(3)}$. Taking the electric field strength $F_{(4)} = dC_{(3)}$, gives a p-brane source called the M2-brane. Taking the Hodge dual of $F_{(4)}$ yields $\tilde{F}_{(7)}$ with 5-dimensional magnetic source called the M5-brane. The spinor is given by a **32**.

Consider the 5-grading of $\mathfrak{e}_{7(-25)}$:

$$\mathfrak{e}_{7(-25)} = \mathbf{1} \oplus \mathbf{32} \oplus (\mathfrak{so}(10,2) \oplus \mathbf{1}) \oplus \mathbf{32} \oplus \mathbf{1}. \tag{1}$$

The 5-grading reveals an $\mathfrak{so}(10,2)$ adjoint representation, as well as its **32** spinor, ingredients suitable for the study of M-theory. What is lacking is the **12** vector of $\mathfrak{so}(10,2)$. In the next section, we show this can be recovered if we use the larger algebra $\mathfrak{e}_{8(-24)}$.

E8 AND M-THEORY

The structure of $D = 10 + 2$ Yang-Mills is almost encoded by the 5-grading of $\mathfrak{e}_{7(-25)}$. To complete the structure, one needs a **12** vector of $\mathfrak{so}(10,2)$.

Consider the following decomposition of $\mathfrak{e}_{8(-24)}$, that makes the 5-grading of $\mathfrak{e}_{7(-25)}$ manifest:

$$\mathfrak{e}_{8(-24)} = 1 \oplus (\mathbf{12} \oplus \mathbf{32} \oplus \mathbf{12}) \oplus [(\mathbf{1} \oplus \mathbf{32} \oplus (\mathfrak{so}(10,2) \oplus \mathbf{1}) \oplus \mathbf{32} \oplus \mathbf{1}) \oplus \mathbf{1}] \oplus (\mathbf{12} \oplus \mathbf{32} \oplus \mathbf{12}) \oplus \mathbf{1}. \tag{2}$$

Symmetries and Order: Algebraic Methods in Many Body Systems
AIP Conf. Proc. 2150, 030010-1–030010-3; https://doi.org/10.1063/1.5124599
Published by AIP Publishing. 978-0-7354-1895-0/$30.00

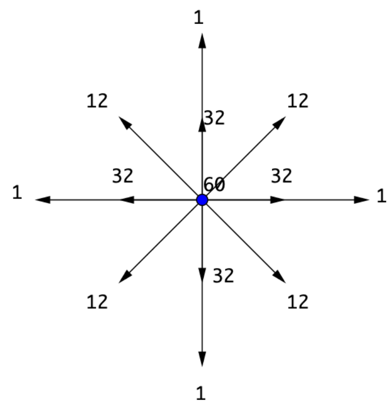

FIGURE 1. A projection of $\mathfrak{e}_{8(-24)}$ along a b_2 (Courtesy of P. Truini).

Figure 1 depicts a projection of $\mathfrak{e}_{8(-24)}$ along a $b2$, giving a visual depiction of the decomposition of $\mathfrak{e}_{8(-24)}$ with manifest $\mathfrak{so}(10,2)$ adjoint, spinor and vector representations. The $\mathbf{12} \oplus \mathbf{32} \oplus \mathbf{12} = \mathbf{56}$ gives a decomposition of the fundamental of $\mathfrak{e}_{7(-25)}$ as a Freudenthal triple system (FTS) in terms of the **12** vector of $\mathfrak{so}(10,2)$.

SPIN FACTORS AND YANG-MILLS

Through the lens of Jordan algebras, the **12** vector of $\mathfrak{so}(10,2)$ can be expressed as:

$$\mathbf{12} = (J_2(\mathbb{O}) \oplus \mathbb{R}) \oplus \mathbb{R} \tag{3}$$

where the $J_2(\mathbb{O})$ spin factor part contributes a $(9,1)$ signature when an element is written with lightcone coordinates on the diagonal. Embedded in $J_3(\mathbb{O})$, an $\mathbb{R}$ is a scalar coordinate from the third diagonal entry of a 3×3 hermitian matrix. The additional copy of $\mathbb{R}$ is the corresponding scalar diagonal of $J_3(\mathbb{O})$ embedded in its FTS, which is the fundamental **56** of $\mathfrak{e}_{7(-25)}$. In terms of signature, the third diagonal of $J_3(\mathbb{O})$ enhances the $(9,1)$ signature to $(10,1)$, and the additional real coordinate gives $(10,2)$ signature.

One can descend to $(10,1)$ by either nullifying a timelike coordinate in the $J_2(\mathbb{O})$ spin factor (which transforms as a **10** of $\mathfrak{so}(9,1)$) or the corresponding diagonal coordinate of the FTS. As $J_2(\mathbb{O})$ can be used to give a matrix formulation of M-theory in $D = 10+1$ in the infinite momentum frame (where the third diagonal coordinate of $J_3(\mathbb{O})$ becomes a point at infinity), we associate the vector space $\mathbf{12} = (J_2(\mathbb{O}) \oplus \mathbb{R}) \oplus \mathbb{R}$ with the total space of M-theory in $D = 10+2$ signature. This is sufficient to show a matrix formulation of M-theory in $D = 10+1$ can be recovered from $\mathfrak{e}_{8(-24)}$ after a timelike coordinate reduction.

ACKNOWLEDGMENTS

The author would like to thank collaborators Piero Truini and Alessio Marrani for inspiration. Moreover, special thanks is given to Sultan Catto and Francesco Iachello for their interest in exceptional periodicity and its magic stars, as well as the invitation to the *Symmetries and Order* symposium in honor of Prof. Iachello (given by Wright Laboratory and the Yale University Physics Department) for which this research letter is dedicated.

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